

Λύσεις Πρώτου Διαγωνίσματος 2/11/2014 Γ Γυμνασίου

γγ11π

Θέμα 1

$$\begin{aligned} \text{Α)} A &= 2^{2000} : [(25^{50} : 5^{99} - 3^{51} : 9^{25})^{1999}] + (2^{111})^{18} - 2 * 2^{1997} = \\ &= 2^{2000} : \left((5^{250} : 5^{99} - 3^{51} : 3^{225})^{1999} \right) + 2^{1998} - 2^{1998} = \\ &= 2^{2000} : ((5^{100} : 5^{99} - 3^{51} : 3^{50})^{1999}) + 2^{1998} - 2^{1998} = \\ &= 2^{2000} : ((5 - 3)^{1999}) + 0 = \\ &= 2^{2000} : 2^{1999} = \mathbf{2} \end{aligned}$$

$$\begin{aligned} \text{Β)} B &= 5\sqrt{\sqrt{16}} - \sqrt{20\sqrt{25}} + \sqrt{2\sqrt{64}} = 5\sqrt{4} - \sqrt{20 * 5} + \sqrt{2 * 8} = \\ &= 5 * 2 - \sqrt{100} + \sqrt{16} = 10 - 10 + 4 = \mathbf{4} \end{aligned}$$

$$\Gamma = \sqrt{\frac{2}{3}} \sqrt{\frac{6}{2}} \sqrt{\frac{8}{4}} = \sqrt{\frac{2*6*8}{3*2*4}} = \sqrt{\frac{96}{24}} = \sqrt{4} = \mathbf{2}$$

$$\Delta = \sqrt{2}\sqrt{3}\sqrt{24} = \sqrt{2 * 3 * 24} = \sqrt{144} = \mathbf{12}$$

Θέμα 2

$$\text{Α)} 4x^2 - \frac{2}{5}x^2 + \frac{1}{2}x^2 = \frac{40}{10}x^2 - \frac{4}{10}x^2 + \frac{5}{10}x^2 = \frac{41}{10}x^2$$

$$\text{Β)} \left(\frac{1}{3}xy\right) (6xy^3) \left(-\frac{1}{2}xy^2\right) = -\frac{1*6*1}{3*2} xxyy^3y^2 = -\frac{6}{6}x^3y^6 = -\mathbf{x^3y^6}$$

$$\text{Γ)} 24x^3y^2 : (-2xy)^3 = 24x^3y^2 : (-8x^3y^3) = -3x^0y^{-1} = -\mathbf{3y^{-1}}$$

$$\text{Δ)} -27\alpha\beta^3(\gamma\chi)^2 : (-3\alpha\chi\beta\gamma)^2 = -27\alpha\beta^3\gamma^2\chi^2 : (9\alpha^2\chi^2\beta^2\gamma^2) = -\mathbf{3\alpha^{-1}\beta}$$

Θέμα 3

$$\begin{aligned} Q(x) &= a(x^3 + x) - 3x^2 + (x - 3)(x^2 + 3x + 9) = \\ &= ax^3 + ax - 3x^2 + x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 = \\ &= (a + 1)x^3 - 3x^2 + ax - 27 \end{aligned}$$

Αλλά,

$$P(x) = 9x^3 - 3x^2 + 8x - 27$$

Άρα πρέπει

$$a + 1 = 9 \Leftrightarrow a = 8$$

Θέμα 4

$$\begin{aligned} P(x) &= (x^3 - 1)^2 + (x^3 + 1)^2 - 2(x^3 - 1)(x^3 + 1) = \\ &= x^6 - 2x^3 + 1 + x^6 + 2x^3 + 1 - 2(x^6 - 1) = 2x^6 + 2 - 2x^6 + 2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} Q(x) &= (x^2 + 1)^3 + 3(x^2 + 1)^2(1 - x^2) + 3(x^2 + 1)(1 - x^2)^2 + (1 - x^2)^3 = \\ &= x^6 + 3x^4 + 3x^2 + 1 + (x^4 + 2x^2 + 1)(3 - 3x^2) + (3x^2 + 3)(1 \\ &\quad - 3x^2) + 3x^4 - x^6 = \end{aligned}$$

$$\begin{aligned} &= 3x^4 + 1 + 3x^4 - 3x^6 + 6x^2 - 6x^4 + 3 - 3x^2 + 3x^2 - 6x^4 + 3x^6 + 3 - 6x^2 \\ &\quad + 3x^4 + 1 + 3x^4 = 1 + 3 + 3 + 1 = 8 \end{aligned}$$