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ΚΕΝΤΡΑ ΟΛΟΚΛΗΡΩΜΕΝΗΣ ΦΡΟΝΤΙΣΤΗΡΙΑΚΗΣ ΕΚΠΑΙΔΕΥΣΗΣ

ΤΣΙΜΙΣΚΗ & ΚΑΡΟΛΟΥ ΝΤΗΛ ΓΩΝΙΑ ΤΗΛ : 270727 – 222594
ΑΡΤΑΚΗΣ 12 Κ. ΤΟΥΜΠΑ ΤΗΛ : 919113 – 949422
www.sygxrono.gr

Επώνυμο: ____________________

Όνομα:____________________

Τμήμα:____________________

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Α Βαθ.	Β Βαθ.	Μ.Ο.

ΔΙΑΓΩΝΙΣΜΑ ΜΑΘΗΜΑΤΙΚΩΝ ΠΡΟΣΑΝΑΤΟΛΙΣΜΟΥ Β’ ΛΥΚΕΙΟΥ

05-01-2017

ΕΝΔΕΙΚΤΙΚΕΣ ΛΥΣΕΙΣ ΘΕΜΑΤΩΝ

ΘΕΜΑ Α

Α1. Απόδειξη σχολικό βιβλίο / σελ 33

Α2. Ορισμός σχολικό βιβλίο / σελ 41

Α3. Λ-Σ-Λ-Λ-Σ

ΘΕΜΑ Β

Β1.

α)

$$\vec{\gamma} = 5\vec{\alpha} - 3\vec{\beta} = 5(1,2) - 3(2,3) = (5,10) - (6,9) = (-1,1)$$

$$|\vec{\gamma}| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

β)

$$\vec{\gamma}\cdot\vec{i} = |\vec{\gamma}|\cdot|\vec{i}|\cdot \sigma\upsilon\nu(\widehat{\vec{\gamma},\vec{i}}) \Leftrightarrow$$

$$\sigma\upsilon\nu(\widehat{\vec{\gamma},\vec{i}}) = \frac{\vec{\gamma}\cdot\vec{i}}{|\vec{\gamma}|\cdot|\vec{i}|} = \frac{(-1,1)\cdot(1,0)}{\sqrt{2}\cdot 1} = \frac{-1+0}{\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} = -\sigma\upsilon\nu\frac{\pi}{4} = \sigma\upsilon\nu(\pi - \frac{\pi}{4}) = \sigma\upsilon\nu\frac{3\pi}{4} \Rightarrow (\widehat{\vec{\gamma},\vec{i}}) = \frac{3\pi}{4}$$

γ)

$$\vec{\delta} \perp \vec{\alpha} \Leftrightarrow$$

$$\vec{\delta}\cdot\vec{\alpha} = 0 \Leftrightarrow$$

$$(\lambda^2\text{-}\lambda,\lambda)\cdot(1,2) = 0 \Leftrightarrow$$

$$(\lambda^2 - \lambda)\cdot 1 + 2\lambda = 0 \Leftrightarrow$$

$$\lambda^2 + \lambda = 0 \Leftrightarrow$$

$$\lambda(\lambda+1) = 0 \Leftrightarrow$$

$$\lambda = 0 \text{ ή } \lambda = -1$$

Β2.

α)

$\overrightarrow{OA}=(2,3)$

$\overrightarrow{OB}=(3,-2)$

$\overrightarrow{OA}\cdot\overrightarrow{OB}=2\cdot 3+3\cdot(-2)=0\Leftrightarrow$

$\overrightarrow{OA}\perp\overrightarrow{OB}\Leftrightarrow\widehat{AOB}=90^0\Leftrightarrow$ ΑΟΒ τρίγωνο ορθογώνιο

$$\left.\begin{array}{l}(OA)=\sqrt{(2-0)^2+(3-0)^2}=\sqrt{13}\\(OB)=\sqrt{(3-0)^2+(-2-0)^2}=\sqrt{13}\end{array}\right\}\Rightarrow (OA)=(OB)\Rightarrow \text{AOB ισοσκελές}$$

β)

Έστω $M(x,0)\in x'x$, τότε $\overrightarrow{AB}=(1,-5)$ και $\overrightarrow{BM}=(x-3,2)$

Τα Α,Β,Μ συνευθειακά $\Leftrightarrow$

$\overrightarrow{AB}//\overrightarrow{BM}\Leftrightarrow$

$\det(\overrightarrow{AB},\overrightarrow{BM})=0\Leftrightarrow$

$\begin{vmatrix}1 & -5\\ x-3 & 2\end{vmatrix}=0\Leftrightarrow$

$1\cdot 2-(-5)(x-3)=0\Leftrightarrow$

$2+5x-15=0\Leftrightarrow$

$5x=13\Leftrightarrow$

$x=\frac{13}{5}$

Άρα $M(\frac{13}{5},0)$

ΘΕΜΑ Γ

Γ1.

$(3\vec{\alpha}-\vec{\beta})\perp(6\vec{\alpha}+5\vec{\beta})\Leftrightarrow(3\vec{\alpha}-\vec{\beta})(6\vec{\alpha}+5\vec{\beta})=0\Leftrightarrow$

$18\vec{\alpha}^2+15\vec{\alpha}\vec{\beta}-6\vec{\alpha}\vec{\beta}-5\vec{\beta}^2=0\Leftrightarrow 18\left|\vec{\alpha}^2\right|+9\vec{\alpha}\vec{\beta}-5\left|\vec{\beta}^2\right|=0\Leftrightarrow$

$5\left|\vec{\beta}^2\right|-9\vec{\alpha}\vec{\beta}-18\left|\vec{\alpha}^2\right|=0\Leftrightarrow 5\left|\vec{\beta}^2\right|-9\left|\vec{\alpha}\right|\left|\vec{\beta}\right|\sigma\upsilon\nu(\widehat{\vec{\alpha},\vec{\beta}})-18\left|\vec{\alpha}^2\right|=0\Leftrightarrow$

$5\left|\vec{\beta}^2\right|-9\cdot 2\left|\vec{\beta}\right|\sigma\upsilon\nu\frac{2\pi}{3}-18\cdot 4=0\Leftrightarrow 5\left|\vec{\beta}^2\right|-9\cdot 2\left|\vec{\beta}\right|(-\frac{1}{2})-72=0\Leftrightarrow$

$5\left|\vec{\beta}^2\right|+9\cdot\left|\vec{\beta}\right|-72=0\Leftrightarrow$

$\left|\vec{\beta}\right|=3$ δεκτή ή $\left|\vec{\beta}\right|=-\frac{48}{10}<0$ απορρίπτεται

Γ2.

Είναι $\vec{\alpha}\vec{\beta} = |\vec{\alpha}||\vec{\beta}|\sigma\upsilon\nu(\widehat{\vec{\alpha},\vec{\beta}}) = 2\cdot 3\cdot(-\frac{1}{2}) = -3$

Οπότε $|3\vec{\alpha}+2\vec{\beta}|^2 = (3\vec{\alpha}+2\vec{\beta})^2 = 9\vec{\alpha}^2 + 12\vec{\alpha}\vec{\beta} + 4\vec{\beta}^2 =$

$= 9|\vec{\alpha}|^2 + 12(-3) + 4|\vec{\beta}|^2 =$

$= 9\cdot 4 - 36 + 4\cdot 9 = 36$

Άρα $|3\vec{\alpha}+2\vec{\beta}| = 6$

Γ3.

$\vec{\alpha}\cdot(3\vec{\alpha}+2\vec{\beta}) = |\vec{\alpha}|\cdot|3\vec{\alpha}+2\vec{\beta}|\cdot\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) \Leftrightarrow$

$3\vec{\alpha}^2 + 2\vec{\alpha}\vec{\beta} = |\vec{\alpha}|\cdot|3\vec{\alpha}+2\vec{\beta}|\cdot\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) \Leftrightarrow$

$3|\vec{\alpha}|^2 + 2\vec{\alpha}\vec{\beta} = |\vec{\alpha}|\cdot|3\vec{\alpha}+2\vec{\beta}|\cdot\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) \Leftrightarrow$

$3\cdot 4 + 2(-3) = 2\cdot 6\cdot\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) \Leftrightarrow$

$12 - 6 = 12\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) \Leftrightarrow$

$6 = 12\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) \Leftrightarrow$

$\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) = \frac{1}{2} \Leftrightarrow$

$\sigma\upsilon\nu(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) = \sigma\upsilon\nu\frac{\pi}{3} \Leftrightarrow$

$(\widehat{\vec{\alpha},3\vec{\alpha}+2\vec{\beta}}) = \frac{\pi}{3}$

ΘΕΜΑ Δ

Δ1.

$\vec{\alpha}\cdot(\vec{\alpha}-2\vec{\beta}) = 5 \Rightarrow \vec{\alpha}^2 - 2\vec{\alpha}\vec{\beta} = 5 \Rightarrow |\vec{\alpha}|^2 - 2\vec{\alpha}\vec{\beta} = 5 \Rightarrow 1 - 2\vec{\alpha}\vec{\beta} = 5 \Rightarrow \vec{\alpha}\vec{\beta} = -2$

$\vec{\beta}\cdot(3\vec{\alpha}+\vec{\beta}) = 10 \Rightarrow 3\vec{\alpha}\vec{\beta} + \vec{\beta}^2 = 10 \Rightarrow 3\cdot(-2) + |\vec{\beta}|^2 = 10 \Rightarrow |\vec{\beta}|^2 = 16 \Rightarrow |\vec{\beta}| = 4$

Δ2.

$\vec{\alpha}\vec{\beta} = -2 \Rightarrow |\vec{\alpha}|\cdot|\vec{\beta}|\cdot\sigma\upsilon\nu(\vec{\alpha},\vec{\beta}) = -2 \Rightarrow 1\cdot 4\cdot\sigma\upsilon\nu(\vec{\alpha},\vec{\beta}) = -2 \Rightarrow \sigma\upsilon\nu(\vec{\alpha},\vec{\beta}) = -\frac{1}{2} \Rightarrow$

$\sigma\upsilon\nu(\vec{\alpha},\vec{\beta}) = \sigma\upsilon\nu\frac{2\pi}{3} \Rightarrow (\vec{\alpha},\vec{\beta}) = \frac{2\pi}{3}$ ή 120^0

Δ3.

$$(3\vec{\alpha}+\vec{\beta})\cdot(\vec{\alpha}-2\vec{\beta})=3\vec{\alpha}^2-6\vec{\alpha}\vec{\beta}+\vec{\alpha}\vec{\beta}-2\vec{\beta}^2=$$

$$=3\left|\vec{\alpha}\right|^2-5\vec{\alpha}\vec{\beta}-2\left|\vec{\beta}\right|^2=$$

$$=3\cdot 1-5\cdot(-2)-2\cdot 16=$$

$$=3+10-32=-19$$

Δ4.

$$\left|x\vec{\alpha}+2\vec{\beta}\right|=7\Rightarrow$$

$$\left|x\vec{\alpha}+2\vec{\beta}\right|^2=49\Rightarrow$$

$$(x\vec{\alpha}+2\vec{\beta})^2=49\Rightarrow$$

$$x^2\vec{\alpha}^2+4x\vec{\alpha}\vec{\beta}+4\vec{\beta}^2=49\Rightarrow$$

$$x^2\left|\vec{\alpha}\right|^2+4x\vec{\alpha}\vec{\beta}+4\left|\vec{\beta}\right|^2=49\Rightarrow$$

$$x^2\cdot 1+4x\cdot(-2)+4\cdot 16=49\Rightarrow$$

$$x^2-8x+15=0\Rightarrow$$

$$x=3 \quad ή \quad x=5$$