

ΔΙΑΓΩΝΙΣΜΑ ΜΑΘΗΜΑΤΙΚΩΝ Γ' ΓΥΜΝΑΣΙΟΥ

02-12-2017

ΕΝΔΕΙΚΤΙΚΕΣ ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ Α

A1.

α) Σ β) Σ γ) Λ δ) Λ ε) Σ

A2.

α) $(3x - y)^2 = 9x^2 - 6xy + y^2$

β) $(a + 5\beta)^2 = a^2 + 10a\beta + 25\beta^2$

γ) $(3a - \beta)(3a + \beta) = 9a^2 - \beta^2$

δ) $(2a + \beta)^3 = 8a^3 + 12a^2\beta + 6a\beta^2 + \beta^3$

ε) $(2 + x)(4 - 2x + x^2) = 8 + x^3$

ΘΕΜΑ Β

B1. α)

$$\begin{aligned} & 2x(x-2)^2 - 3(1-x)^3 + (x+2)(x-2)(x^2+4) = \\ & = 2x(x^2 - 4x + 4) - 3(1 - 3x + 3x^2 - x^3) + (x^2 - 4)(x^2 + 4) = \\ & = 2x^3 - 8x^2 + 8x - 3 + 9x - 9x^2 + 3x^3 + x^4 - 16 = \\ & = x^4 + 5x^3 - 17x^2 + 17x - 18 \end{aligned}$$

β)

$$x = (\sqrt{2} - 1)^3 + (1 - \sqrt{2})^3 - (-1)^{2017} + \left(\frac{25}{32}\right)^0 \Rightarrow$$

$$x = (\sqrt{2} - 1)^3 + [-(\sqrt{2} - 1)]^3 - (-1) + 1 \Rightarrow$$

$$x = \cancel{(\sqrt{2} - 1)^3} - \cancel{(\sqrt{2} - 1)^3} + 1 + 1 \Rightarrow$$

$$x = 2$$

B2. α) $3\alpha^2 - 5\alpha\beta + 7\beta^2 + 3\alpha\beta - 2\beta^2 + 6\alpha - 5\alpha^2 = -2\alpha^2 - 2\alpha\beta + 6\alpha + 5\beta^2$

β) $-4x^3 + 6x + x^2 - 15x^2 + 6x^3 - 4 + x^3 - 15x = 3x^3 - 14x^2 - 9x - 4$

γ) $\cancel{x^3} - 2x + 1 + x^2 - 5x + 7 - \cancel{x^3} + 2 = x^2 - 7x + 10$

ΘΕΜΑ Γ

Γ1. α) $(2x^2 - 3x)(5x^3 + 1) = 10x^5 + 2x^2 - 15x^4 - 3x = 10x^5 - 15x^4 + 2x^2 - 3x$

β) $2x(x + y) - (x + y)(x - 2y) = 2x^2 + 2xy - (x^2 - 2xy + yx - 2y^2) =$

$$= 2x^2 + 2xy - x^2 + 2xy - yx + 2y^2 = x^2 + 3xy + 2y^2$$

γ) $-x^2(x - 2) + x(x^2 - 1) + x(2x + 1) = \cancel{-x^3} + 2x^2 + \cancel{x^3} - \cancel{x} + 2x^2 + \cancel{x} = 4x^2$

Γ2. α)

$$3(2x+1) - 2(3x+1) = 4x - 3 - 4(x-1) \Rightarrow$$

$$\cancel{6x} + 3 - \cancel{6x} - 2 = \cancel{4x} - 3 - \cancel{4x} + 4 \Rightarrow$$

$$0x = -3 - 3 + 4 + 2 \Rightarrow$$

$0x = 0$, αόριστη, έχει άπειρες λύσεις

β)

$$\frac{1-3x}{2} - \frac{x-7}{4} = \frac{x-2}{3} + \frac{1}{12} \Rightarrow$$

$$\cancel{12} \frac{1-3x}{\cancel{2}} - \cancel{12} \frac{x-7}{\cancel{4}} = \cancel{12} \frac{x-2}{\cancel{3}} + \cancel{12} \frac{1}{\cancel{12}} \Rightarrow$$

$$6(1-3x) - 3(x-7) = 4(x-2) + 1 \Rightarrow$$

$$6 - 18x - 3x + 21 = 4x - 8 + 1 \Rightarrow$$

$$-18x - 3x - 4x = -8 + 1 - 6 - 21 \Rightarrow$$

$$-25x = -34 \Rightarrow$$

$$x = \frac{\cancel{-34}}{\cancel{-25}} \Rightarrow$$

$$x = \frac{34}{25}, \text{ μοναδική λύση}$$

ΘΕΜΑ Δ

Δ1. $\alpha \rightarrow 2$ $\beta \rightarrow 1$ $\gamma \rightarrow 4$

Δ2. α)

i.1	ii.3	iii.2
-----	------	-------

β) Ο όγκος του κυλίνδρου είναι:

$$V_1 = \pi r^2 u = 3,14 \cdot 4^2 \cdot 8 = 401,92 \text{cm}^2$$

Ο όγκος του κώνου είναι:

$$V_2 = \frac{1}{3}\pi\rho^2u = \frac{1}{3} \cdot 3,14 \cdot 4^2 \cdot x = \frac{1}{3} \cdot 50,24 \cdot x \quad \text{cm}^2$$

Αφού ο κώνος έχει όγκο το 50% του όγκου του κυλίνδρου, οπότε πρέπει να ισχύει:

$$V_2 = \frac{1}{2}V_1 \quad \text{ή} \quad \frac{1}{3} \cdot 50,24 \cdot x = \frac{1}{2} \cdot 401,92 \quad \text{ή} \quad \frac{1}{3} \cdot 50,24 \cdot x = 200,96 \quad \text{ή}$$

$$\frac{1}{3}x = \frac{200,96}{50,24} \quad \text{ή} \quad \frac{1}{3}x = 4 \quad \text{ή} \quad x = 12\text{cm}$$